Healthcare waiting lists and Linear Programming

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Abstract

The length of healthcare waiting lists represents a critical issue in many countries. This note proposes a Linear Programming model to support decision-making regarding the length of healthcare waiting lists in Italy. Recent investigations on this topic suggest that increasing the supply of healthcare services provided by Local Health Authorities (LHAs) could serve as a key strategy. The proposed model is based on the assumption that *healthcare services can be represented as a "linear combination" of available healthcare resources*. Consequently, it derives from a classic Linear Programming model [5] and can serve as a decision-support tool to optimize the use of an LHA's resources, specifically, by identifying "abundant" versus "scarce" resources and and by evaluating ways to increase the service provision.

1 Introduction

The literature contains many articles on the application of Operations Research to problems in the healthcare sector. In particular, the following link provides references to the annual ORAHS conferences: http://orahs.di.unito.it/

The general framework for such applications is as follows:

(a) mathematically modelling a problem;

(b) solving it to provide decision support.

This note introduces a Linear Programming model for the potential optimal use of resources within a Local Health Authority (LHA) to provide healthcare services that are scheduled via waiting lists.

The Linear Programming (for short, *LP*) model introduced below is an approximation of a real-world scenario; specifically, an Integer Linear Programming model (for short, *ILP*) would be more appropriate, given that resources may consist of indivisible units. However, regarding this aspect, we note the following:

1) solving an *ILP* problem generally entails a much higher computational cost compared to solving the corresponding *LP* problem obtained by removing the integrality constraint on the variables;

2) empirical evidence suggests that for an *ILP* problem with a large number of variables (integer but non-binary), the optimal solution to the corresponding $LP-$ obtained by relaxing the integrality constraint $-$ provides a good approximation of the original problem's optimal solution;

3) the aim here is to introduce a potential decision-support tool, meaning that a good approximation of the original problem's optimal solution may well suffice.

This note proceeds as follows: Section 2 introduces a basic version of the model in order to lay the foundations; Section 3 introduces a general version of the model, showing how the basic version can be adapted and developed to account for several realistic aims and scenarios; Section 4 provides a brief overview of the topic and focuses on the practical use of the model.

2 Basic Model

This section introduces a basic version of the model (i.e., the basic model) in order to lay the foundations.

This basic model derives from the classic *LP* model introduced in [5] for the following scenario: a firm (here, the LHA) has a specific set of resources at its disposal (here, LHA's resources) and can perform a specific set of activities (here, LHA's services). The goal is to maximize the sum of activity levels (here, the number of users served), with the assumption that each activity requires a "linear combination" of resources $-$ an assumption that appears reasonable in the context of the LHA.

The basic model is described below.

Parameters:

:: the chosen time horizon: one week;

:: let $\mathcal{R} = \{R_1, R_2, \ldots, R_n\}$ be the family of LHA's *resources* (human and non-human) of the LHA; that is, each set R_i (for $i = 1, \ldots, n$) represents a resource of the LHA; for example, R_i might represent the set of physiatrists, the set of nurses, the set of abdominal ultrasound machines, etc.;

:: for $i = 1, \ldots, n$, let h_i be the number of *weekly working hours for each element in* R_i; for example, if R_i is the set of physiatrists, then h_i is the number of weekly working hours for a physiatrist;

:: let $A = \{A_1, A_2, \ldots, A_m\}$ be the set of LHA's *services* that the LHA can provide to users and that can be booked via waiting lists; for example, a service might be physiatry consultations, abdominal ultrasounds, blood tests, etc.;

:: for $j = 1, \ldots, m$, let d_j be the *duration of a performance of service* A_j; for example,

if A_j is the service of physiatry consultations, then d_j is the duration of a physiatry consultation;

:: for every pair $(R_i, A_j) \in \mathcal{R} \times A$, let p_{ij} be the *performance coefficient of resource R_i for service A_j*; that is, p_{ij} indicates the fraction of resource R_i required for a user to receive service A_j; specifically, in accordance with the introductory section, providing a patient with service A_j requires a "linear combination" of resources R_i (for $i = 1, \ldots, n$), each with a coefficient p_{ij} (for $i = 1, \ldots, n$); for example, if R_i is the set of physiatrists and A_j is the service of physiatric consultations, then, provided that $|A_i| = 7$, $h_i = 25$ hours, and $d_j = 20$ minutes, it follows that a single physiatrist can perform 75 consultations per week [in fact, $25 \times (60 \ / \ 20) = 75$], and that consequently the coefficient p_{ij} can be computed as $1 \ / \ (75 \times 7) = 1 \ / \ 525$.

Decision variables:

$x_j \quad$ for $j = 1, \ldots, m$;

x_j = performances of service A_j provided by the LHA in a week

Model:

$\mathbf{max} \sum_{j=1,\ldots,m} x_j$

$\sum_{j=1,\ldots,m} p_{ij} x_j \leq |R_i| \quad$ for $i = 1, \ldots, n$

$x_j \geq 0 \qquad$ for $j = 1, \ldots, m$

Comment 1: The first comment concerns the potential validity of the basic model. In general, like all *LP* models, the basic model is quite adaptable to the specifications of a real-world case, which can be numerous.

Specifically, we observe that adding a real-world specification would entail (where possible) the addition of variables and constraints [to the basic model]; assuming the objective function remains unchanged, the basic model would still yield an upper-bound estimate. Similarly, if a resource were proven to be "scarce" for the optimal solution of the basic model, it would be even more likely proven to be "scarce" for the optimal solution of the model obtained by adding variables and constraints [to the basic model].

Let us consider the following example of adding a real-world specification.

Let A_j be the service of intra-articular injections. This service requires a physician. However − and this is the real-world specification − that physician could be either a physiatrist (assumed to be in set R_1) or an orthopedist (assumed to be in set R_2). Consequently, the formulation above could not be implemented, because the "linear combination" for this specific case involves indifference between the use of two resources [meaning they are mutually alternative]. In other words, resources R_1 and R_2 could be viewed EITHER as indispensable and non-alternative resources for certain services, OR as indispensable yet mutually alternative resources for other services. To address this issue, one can proceed as follows: **(a)** introduce variables y_1 and y_2 into the model for resources R_1 and R_2, respectively, when they are considered indispensable but not mutually exclusive, and place variables y_1 and y_2 (instead of $|R_1|$ and $|R_2|$) on the right-hand side of the inequalities associated with R_1 and R_2; **(b)** introduce variable

$y_{1,2}$ into the model for the resources in $R_1 \cup R_2$ when they are considered indispensable but mutually exclusive, and place variable $y_{1,2}$ on the right-hand side of the inequality associated with $R_1 \cup R_2$; **(c)** introduce the constraints $y_1 \leq |R_1|$, $y_2 \leq |R_2|$, and $y_1 + y_2 + y_{1,2} = |R_1| + |R_2|$ into the model.

Comment 2: The second comment concerns the potential utility of the model.

Indeed, the model appears potentially useful as a decision-support tool. Specifically:

:: For each service A_j (where $j = 1, \ldots, m$), one can define a minimum weekly number of performances, namely, a lower-bound (l_j), and a maximum weekly number of performances, namely, an upper-bound (u_j), and then add the constraint $l_j \leq x_j \leq u_j$ to the model. For example, the number u_j could be based on actual user demand, in order to prevent the optimal solution from allocating a high number of performances to low-demand services. However, more generally, such two numbers l_j and u_j allow the decision-maker to define a desired range for performances of service A_j under steady-state conditions.

:: Once the model's optimal solution has been calculated, one can utilize standard *LP* tools, specifically, sensitivity analysis and shadow price analysis (generated by the software); these tools allow resources to be categorized as "abundant" (i.e., non-binding) or "scarce" (i.e., binding) relative to the optimal solution; in the basic model, this assessment would be straightforward; specifically:

:: :: sensitivity analysis for resource R_i (where $i = 1, \ldots, n$): if resource R_i is "abundant", the analysis indicates the limit within which a decrease in R_i does not reduce the optimal solution value; if resource R_i is "scarce", the analysis indicates the limit within which an increase in R_i leads to an increase in the optimal solution value;

:: :: shadow price analysis for resource R_i (where $i = 1, \ldots, n$): in the basic model, this analysis simply indicates whether resource R_i is "abundant" or "scarce".

In summary, consulting these standard tools might reveal, for instance, that the resource of physiatrists is "scarce" and indicate approximately how many additional physiatrists would be needed to ensure that the supply is no longer "scarce".

3 General model

This section introduces a general version of the model (i.e., the general model), showing how the basic version can be adapted and developed to account for several realistic aims and scenarios.

For the sake of simplicity, we introduce only the additional parameters and variables.

Additional parameters:

for $i = 1, \ldots, n$, let c_i be the *weekly cost to the LHA of each element in* R_i; for example, if R_i is the set of physiatrists, then c_i is the weekly cost to the LHA of a physiatrist;

for $j = 1, \ldots, m$, let z_j be the *revenue for the LHA from service* A_j; for example, if

A_j is the service of physiatry consultations, then z_j is the revenue for the LHA from a physiatry consultation.

Additional decision variables:

y_i for $i = 1, \ldots, n$;

y_i = number of elements in R_i utilized by the LHA in a week.

Model:

max $\sum_{j=1,\ldots,m} z_j x_j - \sum_{i=1,\ldots,n} c_i y_i$

$\sum_{j=1,\ldots,m} p_{ij} x_j \leq y_j$ for $i = 1, \ldots, n$

$y_j \geq |R_j|$ for $i = 1, \ldots, n$

$x_j \geq 0$ for $j = 1, \ldots, m$

$y_j \geq 0$ for $i = 1, \ldots, n$

Comment 3: This comment supplements Comment 2 regarding the basic model. Note that at the optimum, the inequalities "$\sum_{j=1,\ldots,m} p_{ij} x_j \leq y_j$" (for $i = 1, \ldots, n$) will be satisfied as equalities, since maximizing the objective function implies that the value of the variables y_i (for $i = 1, \ldots, n$) at the optimum will be as small as possible. As with Comment 2, once the optimal solution indicated by the model has been calculated, one can consult the standard *LP* tools provided by sensitivity analysis and shadow price analysis (computed by software). These tools allow resources to be classified as "abundant" (i.e., non-binding) or "scarce" (i.e., binding) relative to the optimal solution; in the general model, this analysis would be more sophisticated. Specifically:

:: :: sensitivity analysis for resource R_i (for $i = 1, \ldots, n$): if resource R_i is "abundant", this analysis indicates the limit within which a decrease in resource R_i does not result in a decrease in the optimal solution value; if resource R_i is "scarce", this analysis indicates the limit within which an increase in resource R_i results in an increase in the optimal solution value;

:: :: shadow price analysis for resource R_i (for $i = 1, \ldots, n$): in the general model, this analysis does not merely indicate whether resource R_i is "abundant" or "scarce"; it also indicates the following: if resource R_i is "abundant", the analysis indicates the minimum unit selling price for units of R_i that would make such a sale profitable (relative to their use within the optimal solution); if resource R_i is "scarce", the analysis indicates the maximum unit purchase price for new units of R_i that would make such a purchase profitable (relative to their use within the optimal solution).

In summary, consulting these standard tools might reveal, for instance, that the supply of physiatrists is "scarce", indicate approximately how many additional physiatrists would be needed to eliminate this scarcity, and finally specify the maximum purchase price (i.e., hiring cost) for a new physiatrist that would make the hire cost-effective.

4 Concluding remarks

The length of healthcare waiting lists represents a critical issue in many countries, as reported, for example, in [8, 9]. Numerous studies have addressed this problem from various perspectives (e.g., [4, 13]). Regarding the mathematical aspects, the literature heavily relies on methodologies such as Markov decision processes, Linear Programming, and Simulation. Specifically, Integer Linear Programming has proven valuable for supporting decision-making in specific contexts, particularly for the optimal allocation of human and physical resources (see e.g. [7, 10, 12]).

This note, which is a revised version of [6], proposes a Linear Programming model to support decision-making regarding the length of healthcare waiting lists in Italy (see [1] for a recent survey). Recent investigations on this topic [2, 3, 11] suggest that increasing the supply of healthcare services provided by Local Health Authorities (LHAs) could serve as a key strategy. The proposed model is based on the assumption that *healthcare services can be represented as a "linear combination" of available healthcare resources*. Consequently, it is inspired by a classic Linear Programming model [5] and can serve as a decision-support tool to optimize the use of an LHA's resources.

The proposed model can be utilized as follows: once the LHA has estimated a desired performance range for each service A_j under steady-state conditions, the model can be applied to identify "abundant" versus "scarce" resources with respect to these ranges, and to evaluate ways to increase the service provision.

Acknowledgements. Please I would like to witness that just try to pray a lot and am not able to do anything without that - ad laudem Domini.

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